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Contingent Claims Analysis and the Valuation of Pension Liabilities

Tom B.M. Steenkamp

Research Memorandum 1999- 19

Vrije Universiteit Amsterdam

CONTINGENT CLAIMS ANALYSIS AND THE VALUATION OF PENSION LIABILITIES

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The Netherlands'

In this paper, contingent claims analysis (CCA) is used to value pension liabilities for a number of different cases characterized by the nature of the liabilities (nominal, fully or conditionally indexed) and by the possible presence of certain safety covenants in the pension conditions. There is a growing literature about CCA - a technique that defines and analyzes all claims on the company in terms of options. From a CCA viewpoint, the pension claims may be regarded as a form of (corporate) debt for the company or the pension fund. The market value of this security can be divided into a default-risk-free loan minus the value of a written put option. This put option is the price for default risk. Existing option valuation models are used to provide numerical insights into the significance of various factors for the value of pension liabilities. The main factors influencing the value of the pension liabilities are the volatility of the (company's and/or pension fund's) assets, the correlation between liabilities and assets, the relationship between corporate debt and equity, the average duration of the liabilities and the expected inflation. The proposed CCA method is applied to value the pension liabilities and pension surpluses of some Dutch corporate pension funds. The contribution of this paper is to present a novel and unified framework for a (market-related) valuation of pension liabilities. Within this framework some practical problems with existing valuation methods, such as the valuation of conditional indexing clauses and the incorporation of the riskiness of the investment portfolio, can be solved.

(Keywords: Pensions, Economic Value, Contingent Claims Analysis, Option Pricing, Pension Funds, Pension-liabilities)

1. Introduction

The valuation of pension liabilities is a widely debated issue. It is significant for solvency-control by regulatory authorities and in terms of the interactions between the pension fund and the corporation. The methods by which pension liabilities are valued impact on the (optimal) investment policy, as shown by Ezra (1991) and Ambachtsheer (1992). Pension funds may show surpluses or deficits; hence, the valuation of pension liabilities is of relevance for corporate value.

In this paper, pension liabilities are viewed like a (corporate) bond. The cash flows connected with this bond can be likened to interest and redemption payments from capital saved by workers through postponed wage payments. In this paper contingent claims analysis (CCA) is used to value these cash flows. Analysis of pension liability cash flows in this way is **new**; this application is not to be found anywhere else, as yet, in the corporate finance literature.

The origins of CCA can be traced to the seminal Black and Scholes (1973) paper where the authors presented a theory for option valuation and viewed corporate liabilities as combinations of simple option-contracts. Merton (1974) used the principles of option pricing to value corporate bonds. More recently, CCA has been used to value numerous complex **securities**². From a CCA viewpoint, the pension cash flows can be valued with a default-risk free discount rate plus a risk premium. This risk premium is the premium to investors for default-risk. This, in turn, is the chance that the corporation or the pension fund is unable

¹ The remarks and suggestions of Prof. Dr. A. Buckley has lead to significant improvements.

² Important review articles are Smith (1979), Mason & Merton (1983) and Park & Subrahmanyam (1990)

to pay their pension obligations. It can be modelled as a put option - the pension put³. The value of this pension put depends on the terms of the loan (for example, the cash flow pattern, maturity, safety covenants and so on) and the forms, quantities of assets acting as collateral for the pension liabilities.

In this paper, two pension fund models will be used. There may be relevant for different pension cultures such as the United Kingdom and the Netherlands. Figure 1 gives a schematic summary of these models in the form of a balance sheet.

Figure 1 Diagram of two-pension fund models⁴

Model I: stand-alone pension fund

Pension assets	PA	Pension surplus	S
		Pension liabilities	Pl_{st}

Model II: separate pension fund integrated with the corporation

Corporate assets	CA	Shareholders Equity	E
		Corporate Debt	CL
Pension assets	PA	Pension surplus	S
		Pension liabilities	Pl_{int}

The left-hand side of the balance sheets in Figure 1, respectively show the market value of the pension fund on its own and inclusive of corporate assets. The right hand side gives the market value, or economic value⁵, of the liabilities.

In Model I, the pension fund is completely independent of the underlying corporation. The company is not responsible for possible shortfalls in the pension-fund, but has also no automatic rights on possible pension surpluses. In fact the pension fund is an independent financial enterprise. In Model II, there is also a separate pension fund but with formal connections to the underlying company. The shareholders of the underlying company will be the owner of possible pension surpluses and have to pay for possible pension shortfalls. The pension liabilities now have double protection - the corporate and the pension assets.

Within both models three different liability forms are distinguished - nominal, fully indexed and conditionally indexed. In the next part of this paper - section two - nominal, fully indexed and conditionally indexed pension liabilities of a stand-alone pension fund (Model I) will be valued. In section three, the same is done for the pension fund integrated with the underlying corporation. In these sections paragraphs a one-period model methodology is used. In section four, some solutions for the multi-period valuation problem of pension liabilities are suggested. In section five, the contingent claims valuation methodology is applied to value the pension liabilities of some Dutch corporate pension funds. Finally, in section six a summary and conclusions are presented.

³ Treynor (1977) uses this expression.

⁴ The defined symbols are used in the formulas in this paper

⁵ The economic value is in financial-economic terms the best approximation of market value. This is, in our opinion, the CCA value.

2. The Stand Alone Pension Fund

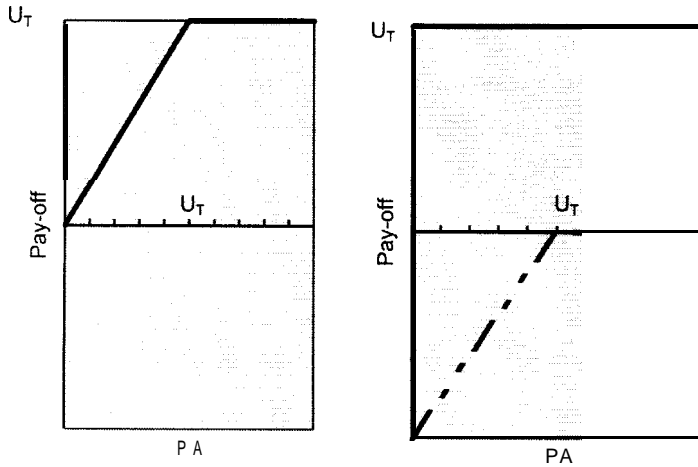
The balance sheet of the stand alone pension fund (see Figure 1) shows, on the assets side, the investment portfolio (pension assets) and on the liability side, the surplus and the pension liabilities. All values are assumed to be market- or economic- values. We will look at a simple one-period model. The pension liabilities have the character of a zero coupon bond with one principal payment at maturity (T). All assets of the fund are collateral for the pension liabilities. When the fund cannot fulfill its obligations, its assets revert to the debt holders.

2.1 Nominal obligations (ABO)⁶

Figure 2 illustrates the pension liabilities pay-off profile at the time of maturity. On the left-hand side, the pay-off for the pension debt holders is outlined at the time of maturity, depending on the market value of the pension assets. The figure shows that the cash flows for the debtholders have option characteristics. The value of the pension liabilities at maturity is the minimum of PA_T or U_T ⁷. On the right hand side of Figure 2, the pay-off profile is given by the nominal redemption value of the pension liabilities (U_T , the solid line) minus the maximum value of $(U_T - PA_T, 0)$ (dashed line). The value of this pay-off profile can, at every point of time t ($t \leq T$), be considered as a combination of a default risk free zero coupon bond B_t ⁸ and a written put option P_t (the pension put). The underlying value of this put option is the value of the pension assets and the strike price is the pension redemption value U_T .

Figure 2 Pay-off profile pension liabilities

Pay-off pension debtholders on T = Default-risk-free payment -/- put-option



⁶ This section draws on Merton (1974).

⁷ U_T is the nominal redemption value of the pension liabilities at T .

⁸ $B_t = U_T \cdot e^{-r_T \cdot T}$ with r_T the nominal default-risk free interest rate for T -periods on the basis of continuous compounding.

The value of nominal pension liabilities in the stand alone pension fund case can be written as shown in equation (1):

$$(1) \quad \begin{aligned} PL_t^{nomsa} &= B_t - P_t(PA_t, U_T) \\ PL_t^{nomsa} &= \text{CCA- valuenominalpensionliabilitis standalonepensionfund} \end{aligned}$$

The pension put is written by the pension debt holders and can be considered as a claim against this group. The value of the put option can be interpreted as:

1. A fair price given to the pension debtholders by the shareholders as a reward for default-risk
2. A price which has to be paid to an insurance company for a loan guarantee. The insurance company will pay when the value of the pension assets, at maturity, is lower then the redemption value of the pension liabilities.

The value of the put option in a one period model can be calculated, using the Black and Scholes option pricing model, as shown in equation (2). We have used a simple version of this formula, without dividend payments and a non-stochastic interest rates.

$$(2) \quad \frac{PL_t^{nomsa}}{B_t} = \{1 - N(-d2^{sa}) + (\frac{1}{w_t^{pa}}) \cdot N(-d1^{sa})\}$$

$$d1^{sa} = \frac{\ln(\frac{1}{w_t^{pa}})}{\sigma_{pa} \cdot \sqrt{T}} + 0,5 \sigma_{pa} \cdot \sqrt{T} \quad d2^{sa} = d1^{sa} - \sigma_{pa} \cdot \sqrt{T} \quad w_t^{pa} = \frac{B_t}{PA_t}$$

Where:

- T = term to maturity of pensionliabilitis andputoption
- σ_{pa} = volatiliy of pensionassets, measured by the standard deviation of returns
- $N(\bullet)$ = cumulative standard normal distribution
- w_t^{pa} = quasi- pensiondebt ratiopensionfund

Equation (2) indicates that the CCA value of pension liabilities as a percentage of the default-risk free value of the nominal pension obligations $(PL_t^{nomsa} / B_t)^{10}$ depends on the term to maturity of the liabilities (T), the volatility of the pension assets and the ratio of the pension liabilities excluding default risk in relation to the total value of the fund, the quasi pension debt ratio $(w_t^{pa})^{11}$.

⁹ These assumptions are not critical for the general methodology, but will influence the numerical outcomes. For a detailed treatment of the different forms of the Black and Scholes formula, see Hull (1994).

¹⁰ B_t is used as a scale factor.

Table 1 is generated from equation (2), the table indicates the CCA pension liabilities value for different (quasi)debt ratios, different portfolio risks measured by the standard deviation of asset returns and different maturities. The table clearly indicates that the CCA value of pension liabilities diminishes if either the quasi debt ratio, the volatility or term to maturity increases.

Table 1 CCA value ($PL^{nom}/B\%$) as a function of volatility, quasi-debt ratio and term to maturity

σ_{pa}	$T \rightarrow$	1	5	10	20	30	40
	$w^{pa} \downarrow$						
	0.2	100	100	100	100	100	99,76
0,1	0,5	100	99,99	99,78	98,38	96,3	94,01
	1	96,0	91,1	87,4	82,3	78,4	75,2
	0,2	100	97,35	88,09	68,41	52,72	40,8
0,4	0,5	99,06	85,01	70,42	50,6	37,6	28,47
	1	84,1	65,5	52,7	37,1	27,3	20,6
	0,2	99,45	77,24	51,88	24,06	11,58	5,71
0,7	0,5	91,92	58,74	36,93	16,35	7,72	3,76
	1	72,6	43,4	26,8	11,8	5,5	2,7

2.2 Fully indexed pension liabilities (PBO or IBO)

We define fully indexed pension liabilities as nominal obligations, which are l_t^{12} adjusted annually to the actual wage or price inflation. The pension liabilities have the character of a zero coupon bond with one cash flow at the term of maturity. Just like the nominal pension obligation the indexed form can be seen as a combination of a default-risk free zero coupon bond and a written put option P_t , as shown in equation (3).

$$(3) \quad PL_t^{real} = B_t^{real} - P_t(PA_t, U_T^{real})$$

$$(4) \quad B_t^{real} = \frac{U_T \cdot (1 + \pi_t^e)^T}{(1 + r_t - rp_t)^T} \approx \frac{U_T}{(1 + real_t - rp_t)^T}$$

where:

¹¹ Merton (1974) introduces the term quasi debt ratio. This ratio can take a value greater than one, because it is calculated with the default risk free value of the pension liabilities.

¹² Unless, of course, the corporation and/or the pension fund go bankrupt.

$PL^{realisa}$ = (current) CCA value of fully indexed pension liabilities of a stand alone fund.

B^{real} = value of default-risk free fully indexed pension liabilities.

π_t^e = expected inflation for the period (t, T) , annualized.

$r_t^{real} = r_t - \pi_t^e$ = ex ante real interest rate for the period (t, T) , annualized.

rp_t = risk premium for unexpected inflation for the period (t, T) , annualized.

One important difference between the nominal and the fully indexed CCA-value is the uncertain value of the pension payment at maturity, due to the future wage and price inflation¹³. This uncertainty has consequences both for the default-risk free value component and the valuation of the option component.

The current CCA value of the fully indexed default risk free pension liabilities B_t^{real} is shown in equation (4). This value differs from its nominal equivalent by virtue of the discount rate. The discount rate consists of the expected real interest rate on time t (r_t^{real})¹⁴ less a risk premium for the unexpected changes in the real rate (rp). Both the real interest rate and the risk premium can change between different maturities. An important practical problem is the determination of this real discount rate. The most obvious method is to derive the discount rate, if existing from marketable index-linked government loans. Other methods can be based on CAPM or APT modelling techniques or, as suggested by Bodie (1976, 1980) or Pesando (1984), by means of a mean-variance analysis¹⁵.

$$(5) \quad P_t = B_t \cdot (1 + \pi^e) \cdot (N(-d2^{real}) \cdot PA + N(-d1^{real}) \cdot U_T^{real}) \quad d1^{real} = \frac{\ln(1/w_t^l \cdot (1 + \pi^e)^T)}{\sigma_{papl} \cdot \sqrt{T}} + 0,5 \sigma_{papl} \cdot \sqrt{T}$$

$$d2^{real} = d1 - \sigma_{papl} \cdot \sqrt{T} \quad \sigma_{papl}^2 = \sigma_{pa}^2 + \sigma_{pl}^2 - 2\rho_{papl} \sigma_{pa} \sigma_{pl}$$

$$(6) \quad \frac{PL_t^{realisa}}{B_t} = (1 + \pi^e)^T \cdot (1 - N(d2^{real})) - \frac{1}{w_t^l} \cdot N(-d1^{real})$$

where:

$\sigma_{pl} = \sigma_{\pi^e}$ = volatility of pension liabilities, measured by the standard deviation of (expected) inflation

σ_{papl}^2 = combined variance of pension assets and pension liabilities

ρ_{papl} = correlation between pension assets and pension liabilities

U_T^{real} = value of fully indexed pension payments at time T

¹³ Actually, the nominal obligations are uncertain too because of the mortality risk. In this paper, we ignore this risk.

¹⁴ In fact the default risk free interest rate.

¹⁵ A detailed analysis for the determination of the real discount rate is, however important, not the subject of this paper

The put option element in the fully indexed value of the pension liabilities is written on the underlying value of the pension assets with a strike price at the indexed pension payments at time T (U_T^{real}). This strike price is stochastic and depends on the uncertain future course of inflation. The standard Black and Scholes valuation is no longer applicable due to this uncertain strike price. Fisher (1978) and Margrabe (1978)¹⁶ have developed option valuation models suited, under certain additional assumptions¹⁷, for the valuation of options with uncertain strike prices. In equation (5), the standard Margrabe formula is rewritten to value a put option with strike price U_T^{real} and underlying value the pension assets. Equation (6) expresses the CCA value of the fully indexed pension liabilities as a percentage of the nominal default risk free pension liabilities.

A comparison of the equations (2) and (6) shows that the main differences between the CCA value of nominal and fully indexed pension liabilities arise from a different measure for the volatility and the influence of expected inflation. The relationship between the volatility of nominal and fully indexed pension obligations can be derived as shown in equation (7).

$$(7) \quad \sigma_{\text{papl}}^2 = \sigma_{\text{pa}}^2 \cdot (1 + \kappa^2 - 2 \cdot \rho_{\text{papl}} \kappa) \quad \text{where } \kappa = \frac{\sigma_{\text{pl}}}{\sigma_{\text{pa}}} \\ \Rightarrow \sigma_{\text{papl}}^2 \leq \sigma_{\text{pa}}^2 \quad \text{if } \kappa \leq 2 \cdot \rho_{\text{papl}}$$

Equation (7) can be interpreted as a sort of **liability hedging credit**¹⁸. The value of the put option is not only dependent on the volatility of the pension assets, but also on the volatility of the liabilities and the connection between the assets and liabilities. If the correlation between pension assets (investment mix) and liabilities is sufficiently positive, the relevant volatility for the option valuation can be lower than the volatility of the ABO liabilities. This also means, all other things equal, a lower put option value.

Clearly, the expected inflation influences the strike price. If those expectations are positive, the strike price level will be higher compared with the strike price of nominal pension liabilities. This means, all other things equal, a higher option value. In general, with positive inflationary expectations, the fully indexed value of pension liabilities will be higher (because of inflation protection) than the nominal variant.

Table 2 is generated by equation (6). The calculations are based on the assumptions that $\sigma_{\text{pl}} = 3\%$, $\rho_{\text{papl}} = 0$ and $\pi^e = 3\%$ ¹⁹. The values in Tables 1 and 2 are directly comparable. Comparison indicates that the fully indexed value is always above the nominal value. The difference between those values diminishes as the default-risk, measured by a higher standard deviation, or a higher quasi debt ratio, increases. Table 2 shows also a value of $\frac{PL_t^{\text{real}}}{B_t}$ greater than 100% if the default risk (low volatility and/or low debt-ratio) is low.

¹⁶ Also known as “exchange-one-asset-for-another-options. See also Hull (1997, p.468).

¹⁷ Additional to the standard Black and Scholes assumptions. The most important additional assumption is the geometric Brownian motion of the strike price (in fact, the inflation).

¹⁸ This term is taken from Sharpe & Tit (1990).

¹⁹ The values for $\sigma_{\text{pl}} = 3\%$, $\rho_{\text{papl}} = 0$ are averages for the period 1976-1996 in the Netherlands. The value for π^e is based on the actual level in 1996.

Table 2 CCA-value of $\frac{PL_t^{realsa}}{B_t}$ as a function of volatility, quasi debt ratio and maturity

σ_{papl}	$T \rightarrow$	1	5	10	20	30	40
	$w^{pa} \downarrow$						
	0,2	103	116,2	135	181,5	235,9	289,4
0,1	0,5	103	116,1	131,9	155,1	169,6	179
	1	97,1	96	96,2	97,1	97,8	98,4
	0,2	103	111,8	112,3	103,4	92,7	82,7
0,4	0,5	101,9	94,5	84,8	70,5	60,1	52
	1	85,3	70,2	60,5	48,6	40,7	34,9
	0,2	102,4	86,5	63,2	34,2	19,1	10,9
0,7	0,5	94,1	64,3	43,6	22,4	12,2	6,8
	1	73,7	46,6	31	15,6	8,5	4,7

2.3 Conditionally indexed pension liabilities

In the foregoing paragraphs CCA valuation was examined with nominal and fully indexed pension liabilities. In practice, fully indexed obligations are rare. The pension scheme often contains, explicitly or implicitly formulated, conditional indexation clauses. These clauses can take different forms. In this paper we assume that the indexation is conditional on the value of the pension assets, PA, a form which is a feature of most Dutch pension schemes. If this is the case, the conditional indexation clause can be formulated as a combination of call options. In a simple one-period model, the pay-off profile of this clause on the time of the pension payment, T, can be described as shown in Table 3 and Figure 3.

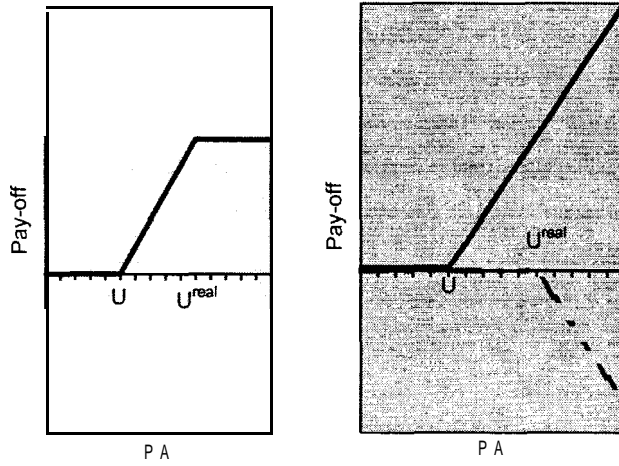
Table 3 Cash flows conditional indexation clause to pension debtholders on time T

	$PA_T \leq U_T$	$U_T \leq PA_T \leq U_T^{real}$	$PA_T \geq U_T^{real}$
Pension debtholders	0	$PA_T - U_T$	$U_T^{real} - U_T$

Table 3 shows that if the value of the pension-assets at time T, namely PA_T , is less then the value of the nominal pension-payments, U_T , the conditional indexation clause yields no cash flow. If the value of the pension asset at time T is not less than the value of the fully indexed pension payments, U_T^{real} , indexation is triggered in full ($U_T^{real} - U_T$). If the value of the pension assets at time T is less then the value of the indexed pension payments but no less the value of the nominal pension payments, indexation is triggered partially equal to an amount of $PA_T - U_T$. This pay-off profile is portrayed on the left-hand side of Figure 3.

Figure 3 Pay-off profile conditional indexation clause

$$\text{Pay-off pension debtholders} = \text{Long call}(PA, U_T) - \text{Short call}(PA, U_T^{\text{real}})$$



The conditional indexation clause pay-off profile can also be written as a combination of a long call option position, with strike price U_T and underlying value PA (the solid line on the right hand side of Figure 3), and a short call option position, with strike price U_T^{real} and underlying value PA (the dashed line on the right hand side of Figure 3).

The value of the conditional indexation clause can be written in equation-form as:

$$(8) \quad CI_t = C_t(PA_t, U_T) - C_t(PA_t, U_T^{\text{real}}) \quad (CI = \text{value of conditional indexing})$$

$$(9) \quad \frac{CI_t}{B_t} = \frac{1}{w_t^{\text{pa}}} \cdot N(d_1^{\text{sa}}) - N(d_2^{\text{sa}}) - \frac{1}{w_t^{\text{pa}}} \cdot N(d_1^{\text{real}}) + (1 + \pi^e)^T \cdot N(d_2^{\text{real}})$$

Equation (8) is the general expression of the conditional indexation clause in terms of call options. Equation (9) expresses the value of the conditional indexation clause relative to the nominal default-risk free value of pension liabilities, using the option pricing models of Black and Scholes and Margrabe, referred to earlier.

We now can express the current value of conditional indexed pension-liabilities as the sum of the value of nominal obligations plus the conditional indexation clause. In the case of a stand alone pension fund, the CCA value of the conditionally indexed pension liabilities is given by the expression (10).

$$(10) \quad PL_t = B_t - P_t(PA_t, U_T) + C_t(PA_t, U_T) - C_t(PA_t, U_T^{\text{real}}) = B_t^{\text{real}} - P_t(PA_t, U_T^{\text{real}})$$

(a)
(b)

Part (a) of equation (10) expresses the value of the nominal pension liabilities, including the default-risk option component. Part (b) expresses the value of the conditional indexation clause. Reformulation of these two parts by means of the put-call parity yields the expression for the fully indexed pension liabilities²⁰. This is an interesting result, because in this case (the stand alone pension fund) there is no difference between the CCA-value of conditional indexed - and fully indexed pension liabilities. This result seems intuitively logical because the value of the collateral (PA) is the same for fully and conditionally indexed liabilities.

3 Pension fund and corporation integrated

The previous section focused upon the valuation of pension liabilities in the stand-alone pension fund case. In this section, we value the liabilities given an explicit relation between the pension fund and the underlying company²¹. The model assumes a separate pension fund. The underlying firm is owner of a possible pension surplus but also has put the corporate assets as collateral for the pension liabilities. In the case of bankruptcy, the pension debtholders have priority above other debtholders and creditors²².

3.1 Nominal and fully indexed pension liabilities

The pay-off profile of the integrated pension fund/corporate model in the case of nominal pension payments is summarised in Table 4²³.

Table 4 Pay-off profile pension debtholders on time T in Model II

$PA_T \geq U_T;$	$PA_T \leq U_T;$	$PA_T \leq U_T;$
$CA_T + PA_T \geq U_T$	$CA_T + PA_T \geq U_T$	$CA_T + PA_T \leq U_T$
U_T	U_T	$(PA_T + CA_T)$

The pay-off at the maturity date now depends on two underlying values, the value of the pension assets PA_T and/or the combined value of pension assets and corporate assets $PA_T + CA_T$. For the sake of convenience, this is defined as the integrated value. If the value of pension assets or the integrated value at time T is no less than the (promised) pension payments, the pension debt holders will be paid fully. If the

²⁰ By means of the put-call parity we can write:

$$C_t(PA_t, U_T) - C_t(PA_t, U_T^{real}) = PA_t + P(PA_t, U_T) - B_t - PA_t - P(PA_t, U_T^{real}) + B_t^{real}$$

²¹ This is model II-type presented in the introduction

²² Non-priority of the pension claim can simply be built into the model. This is not the subject in this paper, but see Steenkamp (1998).

²³ The pay-off profile is given in terms of the nominal pension payments U_T . In the case of fully indexed liabilities, this symbol can be replaced by U_T^{real} .

value of pension assets and the integrated value at time T are less than the (promised) pension payments, the pension debt holders are paid an amount equal to $PA_T + CA_T$. This pay-off profile can be written as:

$$\{U_T \cdot \max[0, U_T \cdot \max(CA_T + PA_T, PA_T)]\}.$$

U_T is the pay-off of a default-risk free zero coupon bond (nominal or fully indexed) with current value B_t , the pay-off $\{\max[0, U_T \cdot \max(CA_T + PA_T, PA_T)]\}$ is equal to the pay-off of a (one-period) put option on the maximum of two risky assets (PA or $PA + CA$), with strike price U_T . This is an exotic option variant known as a two-colour rainbow option. Equation (11) expresses the pay-off profile in terms of the value of the underlying securities at time t ($t \leq T$):

$$(11) \quad PL_t = B_t - P_t^{\text{rainbow}}(PA_t, PA, + CA_t, U_T(U_T^{\text{real}}))$$

P_t^{rainbow} = put- option on the maximum of two risky assets
(two- colour rainbow option)

Stulz (1982) develops a closed form valuation solution for these two-colour rainbow options. The rather complex Stulz formula is applied to this pension model and is expressed in equation (12).

$$(12) \quad \frac{PL_t^{\text{no mint}}}{B_t} = \frac{1}{w_t^{\text{pa}}} \cdot \{1 - N(d - \sigma_{rb} \sqrt{T}) + (1 + gv) \cdot N(d)\} - c_{rb}^{\text{max}}$$

$$c_{rb}^{\text{max}} = \frac{1}{w_t^{\text{pa}}} \cdot \{(1 + gv) \cdot M(y_1, d, \rho_1) + M(d_1^{\text{sa}}, -d + \sigma_{rb} \sqrt{T}, \rho_2)\} + M(-y_1 + \sigma_{\text{capa}} \sqrt{T}, -d_1^{\text{sa}} + \sigma_{\text{pa}} \sqrt{T}, \rho_{\text{capa}}) - 1$$

$$d = \frac{\ln(1 + gv) + T \cdot \sigma_{rb}^2 / 2}{\sigma_{rb} \sqrt{T}} \quad ; \quad y_1 = \frac{\ln(\frac{1 + gv}{w_t^{\text{pa}}}) + T \cdot \sigma_{\text{capa}}^2 / 2}{\sigma_{\text{capa}} \sqrt{T}} \quad ; \quad \rho_1 = \frac{\sigma_{\text{capa}} - \rho_{\text{capa}} \cdot \sigma_{\text{pa}}}{\sigma_{rb}}$$

$$\rho_2 = \frac{\sigma_{\text{pa}} - \rho_{\text{capa}} \cdot \sigma_{\text{capa}}}{\sigma_{rb}} \quad ; \quad \sigma_{rb} = \sqrt{\sigma_{\text{capa}}^2 + \sigma_{\text{pa}}^2 - 2 \cdot \rho_{\text{capa}} \cdot \sigma_{\text{capa}} \cdot \sigma_{\text{pa}}}$$

where:

$PL_t^{\text{no mint}}$ = value nominal pension liability integrated pension fund and corporation (Model II).

$gv = \frac{CA}{PA}$ = market value corporation relative to market value pension fund

$M(.,.,.)$ = value cumulative bivariate Normal distribution

σ_{capa} = volatility corporate and pension assets

ρ_{capa} = correlation pension- and corporate assets

An important difference with the stand-alone case in the previous section is that the value of the pension put option depends on the connection between corporate- and pension assets. If this connection

diminishes, the value of the put option decreases and the value of the pension liabilities increases. Equation (12) shows that compared to the stand alone case three additional factors are important for the valuation of the pension put and thus for the CCA value of pension liabilities. These factors are:

1. The relative size in market value between (underlying) corporation and pension fund,
2. The volatility of corporate assets or corporate value.
3. The correlation between corporate- and pension assets.

We consider each of these factors further in the next several paragraphs.

When the value of the company increases relative to the value of the pension fund, the value of the collateral increases. This better protection means, all other things equal, a lower value of the rainbow put option and a higher value of the pension liabilities.

In general, an increasing volatility, both of corporate assets and pension assets, leads to higher (put) option values. For the rainbow options this will be only the case for non-extreme values.²⁴

If the correlation between corporate assets and pension assets increases, the value of the put option will also increase. In the case of a low or even negative correlation, there will be a high probability that the put option will end up out of the money. The pay-off depends on the maximum of both corporate assets and pension assets. The value of the pension liabilities increases if the correlation increases. This is logical given that there is a high chance that a low value of pension assets goes with a high value of corporate assets, thus giving greater protection against default. This also implies that if the riskiness of the pension assets increases it will be possible that the combined risk of pension assets and corporate assets decreases. Companies like banks and insurers, with fixed income corporate assets characteristics, may actually reduce the riskiness of their total assets by allocating higher percentage shares in their pension fund portfolios. The value of the pension put will decrease and thus the value of the pension liabilities will increase. As an example, assume two different companies, an oil company and a bank. The relevant factors for the valuation of the pension liabilities are given in Table 5.

Table 5 Debt ratio and volatilities for two companies

	Oil company	Bank
Ratio company value / pension fund	0,5	0,5
Coverage ratio pension liabilities	1	1
Volatility corporate assets	0,21	0,10
Correlation assets - equity index	-0,3	0,3
Correlation assets - bond index	0,3	0,9

²⁴ See, for a non-technical explanation, Stulz (1982, par. 3.4.1)

The operational cash flow of the oil company is strongly dependent on the oil price in dollars. The volatility and correlations with respect to the oil company are therefore derived from the oil price. Because the assets of the bank consist mainly of fixed income securities, the volatility and correlation measures are based on a fixed income index. The value of the pension liabilities is calculated with equations (11) and (12) in two cases: 100% fixed income pension assets and a 100% equity pension portfolio. In the 100% fixed income case, the pension liabilities value of the oil company is higher. In the 100% equity case, the pension liabilities value of the bank is higher²⁵.

3.2 Conditionally indexed liabilities

We assume the same conditional indexing clause as in section 2.3. We can express the value of the conditional indexed pension liabilities as a combination of the nominal value (including default risk) and the value of the conditional indexation clause as written²⁶ in equation (13):

$$(13) \quad PL_t = B_t - P_t^{\text{rainbow}}(CA_t + PA_t, PA_t, U_T) + C_t(PA_t, U_T) - C_t(PA_t, U_T^{\text{real}})$$

Compared with the stand-alone case, the value of conditionally indexed pension liabilities will deviate from the value of fully indexed and nominal liabilities. Numerical values can be calculated if we combine equations (12), the closed form equation for nominal pension liabilities in the integrated case and equation (9), the closed form solution for the conditional indexation clause.

4 Valuation in a multi period model

The description of the capital structure in general, or the pension liabilities in particular, in a one period model is an important abstraction of reality, but it can be justified by some clear advantages. An important advantage is the availability of closed form option pricing formulas. The relation between input factors and option value is therefore specified. In practice, pension liabilities have multi period cash flows, each individually contains a put option element. In order to use our one-period abstraction we have to bundle a multi period cash flow structure into a one-period payment. A simple method is to take some average characteristics (age, years of employment, salary) of the pension debtholders of a particular pension scheme. Another method is to calculate the duration of the pension cash flows. Duration functions, in a one-period model, as a measure of the term to maturity for the pension put-option and pension liabilities. In this manner, we can, analytically, put corporations with different pension characteristics into one framework. This method will be used to determine, empirically the value of pension liabilities for different Dutch corporations in section five.

²⁵ Exact calculations are available, on request, from the author.

²⁶ An important problem, with the expression of the conditional indexing clause, is the case of deflationary expectations. In this case, the indexed payments will be lower than the nominal pension payments. In this case, the value of the call option with (uncertain) indexed strike price, U_T^{real} , can be higher than the value of the call option with nominal strike price U_T . Due to this fact, the value of the conditional indexing clause can be negative. To avoid this property, we have to add an extra restriction: if $U_T^{\text{real}} \leq U_T$ the conditional clause value is zero.

The valuation of multi period pension liabilities is also possible, and theoretically more appropriate, with CCA-models for the valuation of corporate coupon bonds. The pension liabilities can, in this case, be considered as a combination of a default risk free coupon bond and a compound put option. In the literature numerical methods are applied to solve the valuation problem for corporate coupon bonds²⁷. In general, there are no closed form analytical solutions for these compound options, with the exception of Longstaff and Schwartz (1996). The authors take the Black and Cox (1976) model as a starting point. With their method, the value of the multi period pension cash flows can be considered as the sum of the individual parts. This means that every individual cash flow in time can be valued as separate zero coupon bonds with (one-period) put options²⁸.

5 The CCA-value of some Dutch pension funds

In this section, we try to determine the CCA-value of pension liabilities and pension surpluses of some Dutch pension funds. The selection of pension funds is based on a recent study by Merrill Lynch (1998). In their study, pension figures of companies, which are constituents of the AEX index, were collected and analyzed. These collected data form the basis of our study and comprise the following: balance sheets of pension funds (end of 1996), company figures (market capitalization and enterprise value) and the pension fund investment mix (end of 1996) between broad asset classes.

We assume that the pension funds have the character of our model II pension fund (integrated with the underlying corporation). In the Netherlands, assets and liabilities of the company pension fund are strictly separated from the company balance sheet. There is, however, no legal objection to transferring surpluses or to reducing premium contributions. On the other hand, there is often no legal duty to underwrite the pension fund in the case of a shortfall²⁹. The liabilities are conditional indexed in line with the conditional indexed clause defined in section 2.3. Given these assumptions we can use the formulas (12) and (9) to calculate the values of the pension liabilities. From these, we can also derive the following data needs:

1. *The (average) term to maturity or duration³⁰ of the pension liabilities (T)*. These duration values have to be derived from the different cash flow characteristics of the pension liabilities. Liability calculations have been made for different durations. We take values of 10, 15 and 20 as an approximation to a young, average and "old" pension fund.³¹
2. *The value of nominal default risk free pension liabilities (B)*. This variable may be relatively simple derived from the Merrill Lynch figures for different duration values. These reported pension liability figures, presented in column 1 of Table 6, are, in general, based on the annual reports. Calculations have been done in general, with a discount rate of 4%. If we wish to use another discount rate we can recalculate the pension liability value with the following formula:

²⁷ See, for example, Kim, Ramaswamy and Sundaresan (1993).

²⁸ This method is further exemplified in Steenkamp (1998).

²⁹ These points are disputable. We think that model II is a reasonably approximation of the majority of pension schemes in the Netherlands.

³⁰ In section four, the duration measure was suggested as a one-period approximation for the multi period character of the pension cash flows.

³¹ These values are based on non-public ALM studies and through simulation experiments.

$$(14) B_t(R\%) = \frac{PL(4\%)}{(1 + R - 0.04)^{Duration}}$$

With this approach, we can recalculate the Merrill Lynch data for pension liabilities values. To determine the value of the nominal default-risk free pension liabilities (B) the yield on long term government bonds ultimo 1996, 5.7%, is used³².

3. *The market value Of the pension assets.* These data were given in the Merrill Lynch paper.
4. *The market value of the underlying corporation.* These values were approximated by the enterprise values used in the Merrill Lynch paper³³.
5. *Volatilities and correlations.* Pension assets were divided in broad asset classes • equity, bonds, short term fixed income and property. The volatilities of these classes were calculated on the basis of long term historical return data on class indices. The same was done for the correlations among the asset classes and the correlation with inflation. The volatility of pension assets was calculated on the basis of these figures³⁴. The volatilities of corporate assets were approximated by the volatility of equity returns of the individual company.
6. *Expected inflation.* Because no index-linked bonds are available in the Netherlands this is one of the most subjective figures. We have used the actual inflation in December 1996, 2.5%, as a measure of (long-term) inflationary expectations. The value of the pension liabilities was also be calculated on the basis of 2% and 3% expected inflation.

The results of the calculated CCA pension liability values are presented in Table 6. The second column gives the reported (book) value at the end of 1996. The third column gives the percentile difference between the reported book value and the calculated CCA value, based on an expected inflation of 2.5% and a duration of 15 years. The third and fourth columns gives the percentile difference between the reported book value and the calculated CCA value, based on durations of 20 years (“young pension fund”) and 10 years (“mature pension fund”).

On average the calculated CCA value (in column three) of the pension liabilities lies 1.2% above the reported book value. The highest differences are Hagemijer, a 10.3% higher CCA value then reported, and KBB, a 12.2% lower CCA value than reported. If we select two groups on the basis of a decreasing order in the difference between CCA value and reported book values, the following statements can be made. The group with the highest differences between market-value and book value has, on average, a higher coverage ratio, lower volatility of the pension assets, a higher relative size of the underlying corporation and lower correlations between respectively corporate assets and pension assets and between pension assets and inflation.

³² As an example we take the reported (book) value of the pension provision of ABN AMRO. The reported value (for example, PL(4%) in formula (13)) was, at the end of 1996, 6.5 bln guilders. The nominal market value of the default-risk free pension provisions, calculated at a discount rate of 5.7% and a duration of 15, was 5.1 bln guilders.

³³ The enterprise value is defined as market capitalization plus net debt.

³⁴ The Merrill Lynch paper also contains data on the division of pension assets into broad asset classes.

Table 6 CCA-pension liabilities values of Dutch company pension funds

Company	PL(4%) (reported Value)	D=15 Infl = 2.5%	D=20 Infl=2.5%	D=10 Infl=2.5%
Philips	19852	2.80%	2.56%	0.21%
Royal Dutch	15747	1.50%	0.40%	0.69%
Klm	7946	5.50%	5.91%	-0.57%
Abn Amro	6474	6.70%	7.24%	-0.88%
Hoogovens	5312	0.90%	0.60%	0.35%
Akzo Nobel	5261	-1.70%	-2.76%	1.18%
Dsm	4451	1.70%	1.42%	0.35%
Unilever	4192	6.90%	7.13%	-0.78%
Stork	2294	-2.70%	-3.49%	1.08%
Heineken	1750	3.60%	3.14%	0.11%
Nedlloyd	1649	6.50%	7.16%	-0.85%
Ahold	1251	4.90%	4.72%	-0.30%
Kbb	918	-12.20%	-14.81%	4.09%
Knv bt	806	7.40%	8.06%	-1.04%
Vws	716	1.00%	0.56%	0.41%
Internatio M.	605	6.80%	7.44%	-0.77%
Wolters Kluwer	599	6.20%	6.84%	-0.94%
Oce	495	-0.10%	-1.01%	0.81%
van Ommeren	405	2.00%	1.98%	0.24%
Elsevier	400	5.40%	5.75%	-0.71%
Hagemeyer	364	10.30%	11.81%	-1.75%
Csm	312	7.90%	8.97%	-1.48%

Because of the uncertainty of some data values a sensitivity analysis was run for the four largest pension funds in the dataset. Table 7 gives the percentile difference between CCA values for different scenarios compared with the base case • a lower risk free rate, lower share prices, higher asset volatilities, higher correlations between assets and inflation, a lower expected inflation and lower size of the underlying company.

Table 7 Sensitivity analysis CCA values Dutch companies

	Base case	Risk free rate -1%	Equity index 20%	Volatility + 10%	Correlations +25%	Inflation -1%	size -50%
Philips	20409	7.32%	-16.76%	9.82%	1.02%	-10.07%	11.11%
Royal Dutch	15985	9.05%	-15.58%	6.25%	1.08%	-8.76%	8.90%
Klm	8383	7.52%	-16.75%	9.54%	0.96%	-10.57%	11.76%
Abn Amro	6908	8.40%	-16.11%	7.64%	0.96%	-10.52%	11.54%

The results in Table 7 indicate that the main influential factors are the risk free real rate and the level of share prices.

6 Summary and conclusions

In this paper, we have focused upon the valuation of pension liabilities. A consistent, objective method for liability valuation is developed on the basis of corporate finance principles and option pricing theory. An important point about this method is that it provides a means for valuation of provisional indexation, and furthermore, it can handle the effects of all kinds of company commitments, such as the (priority) treatment of pensions in cases of bankruptcy and the value of certain safety covenants on the value of the pension liabilities.

Pension liabilities are regarded as a form of corporate debt, which can be valued by means of contingent claims analysis (CCA). From the CCA viewpoint, the risk that a pension fund will not be able to meet its pension liabilities can be valued as a put option, the pension put. While the pension put can be unambiguously interpreted, it can assume various forms (including a number of exotic types). For example, the situation when pension liabilities are not given priority in case of bankruptcy can be modelled as a barrier option, while fully indexed liabilities can be modelled as a rainbow option. The value of the pension put also depends on how the collateral for the pension liabilities is arranged.

Different forms of pension liabilities are theoretically derived and numerically illustrated in this paper. In Table 8 the main results of the paper are summarized in terms of the relation between the CCA value of different forms of pension liabilities (PL) as a percentage of the default-risk free value (B) and the different value factors.

Table 8 *Relation between pension liability value and explanatory variables*

PL/B →	Standalone Nominal	Standalone (Conditional) Indexed	Integrated Nominal	Integrated Fully indexed	Integrated Conditionally indexed
PA/B	+	+	+	+	+
CA/PA	0	0	+	+	+
Duration	•				
σ_{pa}					-/+
σ_{ca}	0	0			-/+
σ_{infl}	0	-/+	0	-/+	-/+
ρ_{painfl}	0	+	0	+	+
ρ_{capa}	0	0			
π^e	0	+	0	+	+

* 0 means no connection, + means ≥ 0 , - means ≤ 0 .

We have identified nine factors which are important for the value of a number of pension liabilities . the coverage ratio of the pension fund (PA/B)³⁵, the size of the pension fund relative to the size of the underlying company (market values, CA/PA), the duration of the pension liabilities (T), volatilities of pension assets and corporate assets and inflation (σ_{pa}, σ_{ca} and σ_{infl}), correlations between, respectively, pension assets and inflation and pension assets and corporate assets (ρ_{painfl} and ρ_{capa}) and, finally, expected inflation (π^e). Most of these value-factors seem intuitively logical. Also asset and liability analysis, using stochastic simulation techniques, conclude that these factors play an important role.

The CCA approach is aimed at the determination of the *economic value* of a security. In an efficient market this value can be regarded as the price the pension liabilities yield when the owners of this security (the past and present employees) sell it on the secondary market. Knowledge of this economic value and the influencing factors can be important for different purposes. This applies, first, with respect to the judgement of the financial position and financial policy of the company and its pension fund. It is shown in section five that book value measures of the pension liabilities can be very different from the calculated CCA values. At the same time, CCA value concepts can be used to determine better estimations of pension costs. Second, the CCA concept can play an important role in the context of current valuation practice and the monitoring by regulatory authorities. The value of the pension put option can be used by the regulator for adequate assessment of the company's pension fund capital position, since this value can be regarded as the loan guarantee premium which would have to be paid to the authorities or a (re)insurer to provide a guarantee that the pension payments will be met in the future. From the viewpoint of the regulator, the value of the put option is an insurance premium, which would have to be paid from the pension surplus to guarantee the pension liabilities against default-risk. Thus, the real value of the pension fund's assets is equal to that of the surplus in the absence of default risk, minus the value of the pension put³⁶.

A final application of CCA valuation is the analysis of the optimal funding and allocation policy of the pension fund. The main conclusion in the literature, based on Sharpe (1976), Harrison and Sharpe (1983) and Black (1980), is that on the basis of combined effects of taxes and the pension put, the optimum funding and asset allocation policy will always be a corner solution with either maximum or minimum funding and either 100% bonds or 100% shares in the portfolio. Steenkamp (1998) has shown, on the basis of the CCA valuation method in this paper that the effects of taxes, pension put and conditional indexing will lead to the conclusion that an optimum funding level and an optimum investment mix other than the corner solution can be found.

³⁵ In this paper, the reverse of the coverage ratio is used, the quasi pension debt ratio w_t^{pa} . This is in line with papers on the valuation of corporate debt.

³⁶ This method is considered in detail in Steenkamp (1998) and is the subject of a forthcoming paper.

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